KJ-1310

B.Sc. (Part - II)
Term End Examination, 2020

MATHEMATICS

Paper - II

Differential Equations

Time : Three Hours] [*Maximum Marks* : 50

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Find the series solution of the linear differential equation

$$4xy'' + 2y' + y = 0.$$

(*b*) Find the Wronskian of J_n and J_{-n}.

(c) Show that $\left(1-2xz+z^2\right)^{-\frac{1}{2}}$ is a solution of the equation

$$z\frac{\partial^2(zv)}{\partial^2}+\frac{\partial}{\partial x}\left\{\left(1-x^2\right)\frac{\partial v}{\partial x}\right\}=0$$

Unit-II

2. (a) If $F(t)$ is a function of class A and if

$L\{F(t)\}=f(p)$, then

$$L\{t^n F(t)\}=(-1)^n\frac{d^n}{dp^n}f(p)$$

where $n = 1, 2, 3, ...$

(b) Apply convolution theorem to prove that

$$B(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}\ ,$$

$m > 0,\ n > 0.$

(c) Solve $Dx + Dy = tD^2x - y = e^{-t}$, $t > 0$ if $x(0) = 3$, $x'(0) = -2$, $y(0) = 0$.

Unit-III

3. (*a*) Solve : $p \cos (x + y) + q \sin (x + y) = z$

(*b*) Solve : $x^2p^2 + y^2q^2 = z^2$

(*c*) Solve by Charpit's method

$$px + qy + pq = 0$$

Unit-IV

4. (*a*) Classify the following P.D.E. and reduce to cannonical form $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$.

(*b*) Find the general solution of the differential equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \sin ny .$$

(*c*) Solve : $r + (a + b) s + abt = xy$

Unit-V

5. (*a*) Find the value of the curve $y = f(x)$ which corresponds to the extremum of the functional $I\left[y(x)\right] = \int_{x_1}^{x_2} x^n \left(\frac{dy}{dx}\right)^2 dx$ and passes through two fixed points $x = x_1$ and $x = x_2$.

(*b*) Find the shortest distance between the parabola $y = x^2$ and the straight line $x - y = 5$.

(*c*) Find the extremum of the functional $I = \int_{x_1}^{x_2} \left(y'^2 + z'^2 + 2yz\right) dx$ with $y(0) = 0$, $z(0) = 0$ if the point (x_2, y_2, z_2) moves over the fixed plane $x = x_2$.

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